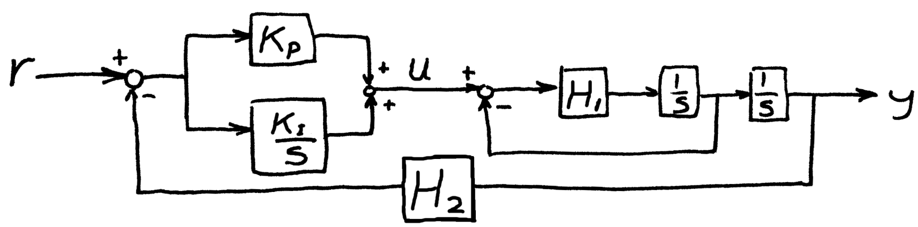
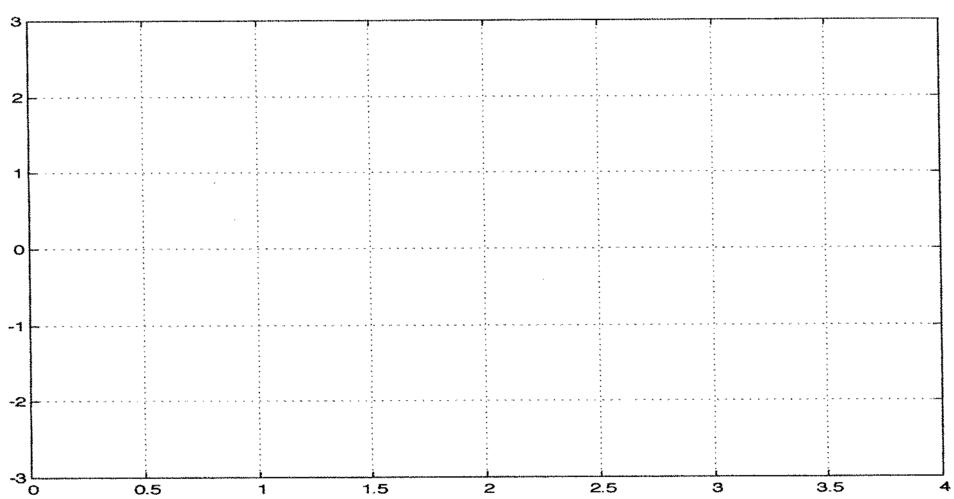
ME 132, Fall 2001
Test Number 2, November 19

Remember to show all your work and box your final answers. **Don't** just write an answer down with no surrounding discussion or equations. If you write something that's incorrect and don't cross it out you won't be likely to get partial credit. All problems are worth 10 points. Good luck.

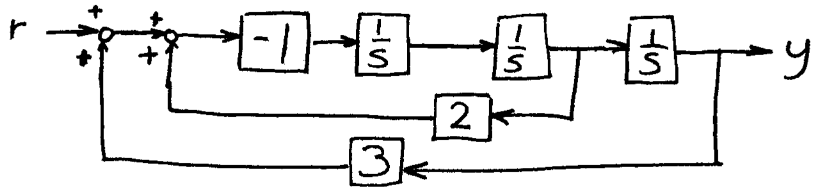
1. (a) Find the transfer functions $\frac{Y(s)}{R(s)}$ and $\frac{U(s)}{R(s)}$ for the illustrated system. K_I, K_P, H_1, and H_2 are constants.
   (b) Find the ordinary differential equation represented by $\frac{Y(s)}{R(s)}$.
   (c) Assume that $r(t)$ is a step input $5h(t)$ (where $h(t)$ is a unit step function. Initially y and all of its derivatives are zero. What are $y(0^+)$, $\dot{y}(0^+)$, $y(\infty)$ and $\dot{y}(\infty)$ equal to?



2. Determine the values of a_1 and ϕ in order to express $x(t) = \sin(\frac{2\pi}{3}t) - \sqrt{3}\cos(\frac{2\pi}{3}t)$ as $a_1 \sin(\pi t - \phi)$. Make a careful sketch in the graph below of what $x(t)$ will look like.



3. Is the system shown below a stable one?

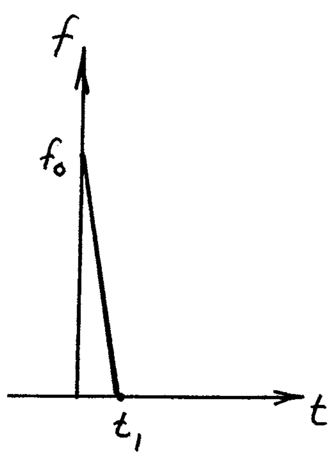


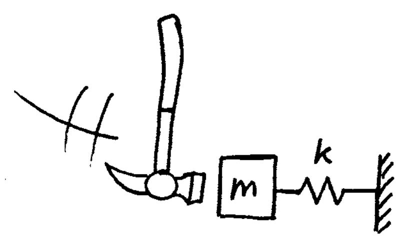
4. What is the magnitude and phase of

$$G(s) = \frac{2\sqrt{3}(2+s)}{(s^2 + \frac{1}{2}s + 5)(\frac{2}{\sqrt{3}} + s)}$$

for $s = 2i$?

5. Recall how I mentioned that impulse hammers work by imparting what looks like a delta function to the structural system under examination. Such a hammer strikes the (initially at rest) mass as shown below. The force/time input is shown as well. Use what you know about Dirac delta functions to determine (approximately) what m's position, velocity and acceleration are at $t = t_1$ in terms of the illustrated parameters. Assume that $t_1 << \frac{2\pi}{\omega_n}$ (where ω_n is the spring/mass system's natural frequency).

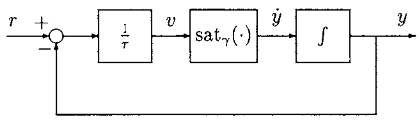




6. Recall the second problem of Homework 7:

In simple mathematical models for actuators (like the motor/aileron control surface on an airplane), often we would like to capture the fact that the output of the actuator (the angular position of the flap) cannot move faster than a certain limit, but otherwise responds like a first-order system.

(a) A block diagram of a model for this behavior is shown below. Assume that $\tau > 0$. The saturation block has the static (nondynamic) relationship shown in its block - the output is the input for small inputs, but beyond a certain range, the output is limited.



$$\mathrm{sat}_\gamma(v) = \begin{array}{rl} \gamma & \text{if } \ v > \gamma \\ v & \text{if } \ -\gamma \leq v \leq \gamma \\ -\gamma & \text{if } \ v < -\gamma \end{array}$$

There are actually three (3) different regimes in which this model behaves differently. For each of the cases below, explain what $\dot{y}(t)$ is in each regime:

i. What is $\dot{y}(t)$ when $r(t) - \tau\gamma < y(t) < r(t) + \tau\gamma$
ii. What is $\dot{y}(t)$ when $y(t) < r(t) - \tau\gamma$
iii. What is $\dot{y}(t)$ when $r(t) + \tau\gamma < y(t)$
iv. Define $V(t) := [r(t) - y(t)]^2$. Suppose $r(t)$ is constant, $\bar{r}$. Show that regardless of what regime we are in, $\dot{V}(t) \leq 0$, and in fact, $\dot{V}(t) = 0 \Leftrightarrow y(t) = \bar{r}$, and $y(t) \neq \bar{r} \Leftrightarrow \dot{V}(t) < 0$.

Assume that the input to the system is given by $r(t) = 4h(t)$ where $h(t)$ is a unit step function. Are we guaranteed that eventually $y(t)$ will approach 4 or might there be a steady-state error? $\tau = 1$ and $\gamma = 2$.