UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME132 Dynamic Systems and Feedback

Midterm Examination II **Spring 2007**

Closed Book and Closed Notes. Six 8.5×11 pages of handwritten notes allowed.

Your Name:

Please answer all questions and draw a box around your final answer for each question.

Problem:	1	2	3	Total
Max. Grade:	20	40	40	100
Grade:				

1 Problem

A dynamic system is described by

$$Y = \frac{(4 - 4\,s)}{(s+2)^2}\,U$$

where $u(t)$ is the input and $y(t)$ is the output.

(a) Write the ordinary differential equation (ODE) that $y(t)$ satisfies.

(b) Obtain the general homogeneous solution $y_H(t)$ of the ODE.

(c) Assume now that the initial conditions at $t = (0^-)$, are all zero, i.e. $y(0^-) = \dot{y}(0^-) = 0$ and that the input is a unit step, $u(t) = 1(t)$.

 (i) Determine the initial conditions at $t = (0^+)$, namely $y(0^+)$ and $\dot{y}(0^+)$.

 (ii) Determine the steady state output, $\bar{y} = \lim_{t\to\infty} y(t)$ and verify that $\bar{y}$ is a particular solution to the ODE for $t > 0$.

 (iii) Obtain the solution $y(t)$ of the ODE and sketch it. Be sure to indicate important information on the sketch such as initial slope, final value, zero crossing, settling time, etc.

2 Problem

Figure 1 shows the block diagram of a control system for an inverted pendulum like system.

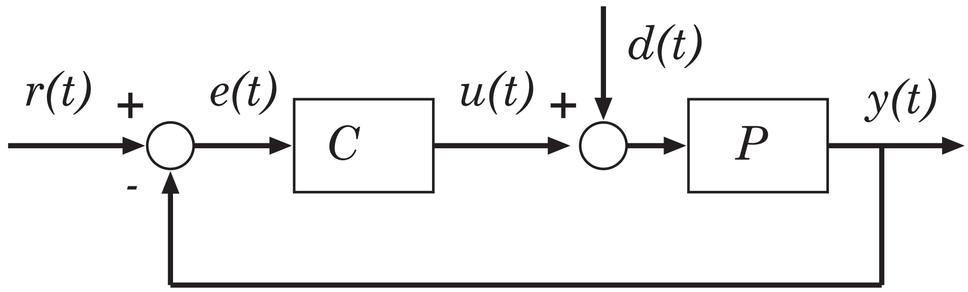

Figure 1: Feedback System

where $r(t)$ is the reference input, $d(t)$ is the disturbance input and $y(t)$ is the output. The plant is open loop unstable and has the following transfer function

$$P(s) = \frac{1}{s^2 - 1} = \frac{1}{(s+1)(s-1)} \tag{1}$$

(a) Assume that a proportional controller is used

$$C(s) = K_p$$

Obtain the close loop characteristic polynomial and determine if the close loop system can be made asymptotically stable with proportional action.

(b) Assume now that a cascade PD controller is used

$$C(s) = K_P + K_D\, s$$

Obtain the close loop characteristic polynomial and determine the range of values of the control gains K_P and K_D so that the close loop system is asymptotically stable.

(c) Determine the values of the control gains K_P and K_D so that the close loop poles will have a natural frequency $\omega_n = 4$ rad. and a damping ration $\zeta = 0.707$. Also, for these values of K_P and K_D:

(i) Find the close loop transfer function form r to y, $G_{R\to Y}(s)$.

(ii) Find the close loop transfer function from d to y, $G_{D\to Y}(s)$.

(iii) Determine the steady state output $\bar{y} = \lim_{t\to\infty} y(t)$ of the close loop system for unit step reference and disturbance inputs, i.e. $r(t) = 1(t)$ and $d(t) = 1(t)$.

(iv) Draw the Bode gain and phase asymptotes of the open loop transfer function $L(j\omega) = C(j\omega)P(j\omega)$ in the graphs shown in Fig. 2.

(v) Sketch the Nyquist plot of the open loop transfer function $L(j\omega)$.

3 Problem

Consider again the feedback system in Figure 1. In this case the plant transfer function $P(s)$ is the same as in Eq. (1) (which is open loop unstable), but the controller is a cascade PID

$$C(s) = K_P + \frac{K_I}{s} + K_D\, s$$

with gains K_P, K_I and K_D selected so that the close loop system is asymptotically stable.

- Figure 3 shows the Bode plot of the open loop transfer function $L(j\omega) = C(j\omega)P(j\omega)$.
- Figure 4 shows the Bode plots of the close loop transfer functions $G_{R\to Y}(j\omega)$ and $G_{D\to Y}(j\omega)$.

(a) Determine the gain crossover frequency, ω_g and the gain margins $(\underline{\gamma}, \overline{\gamma})$ at the control input location. The answer should not be given in decibels.

(b) Determine the phase phase crossover frequency, ω_p, the phase margin and the time delay margin at the control input location.

(c) Determine the steady state output $\bar{y} = \lim_{t\to\infty} y(t)$ of the close loop system for unit step reference and disturbance inputs, i.e. $r(t) = 1(t)$ and $d(t) = 1(t)$.

(d) Determine the steady state output, $\bar{y}(t) = \lim_{t\to\infty} y(t)$ of the close loop system for:

- a unit step reference input, $r(t) = 1(t)$ and
- a sinusoidal disturbance input, $d(t) = 10\, \sin(2\, t)$.

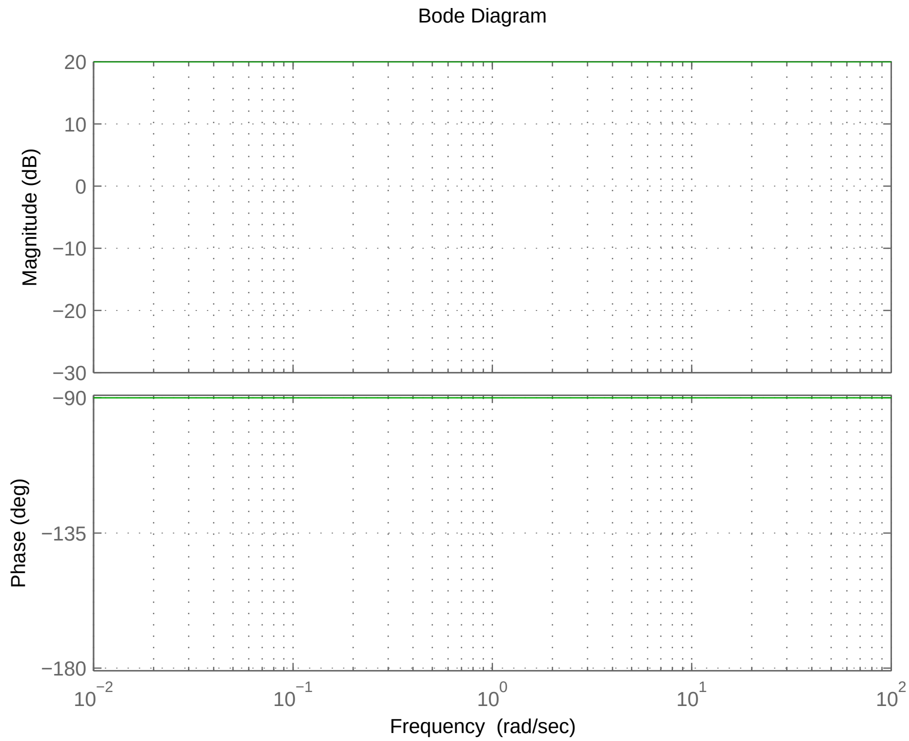

Figure 2: Draw the Bode plot of $L(j\omega)$ in problem 2 here.

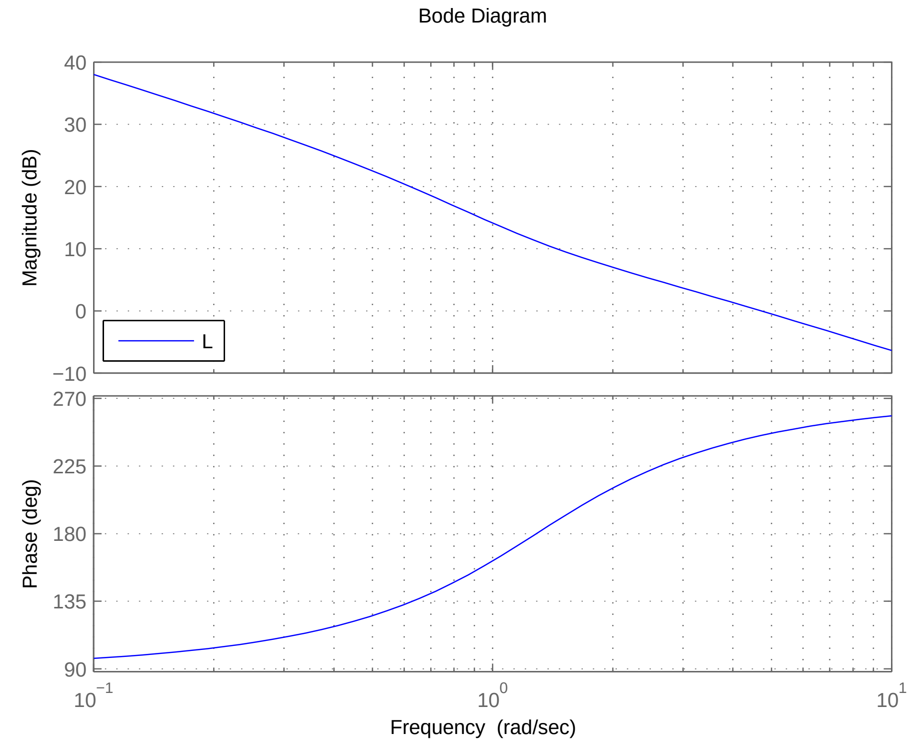

Figure 3: Bode plot of $L(j\omega)$ in problem 3.

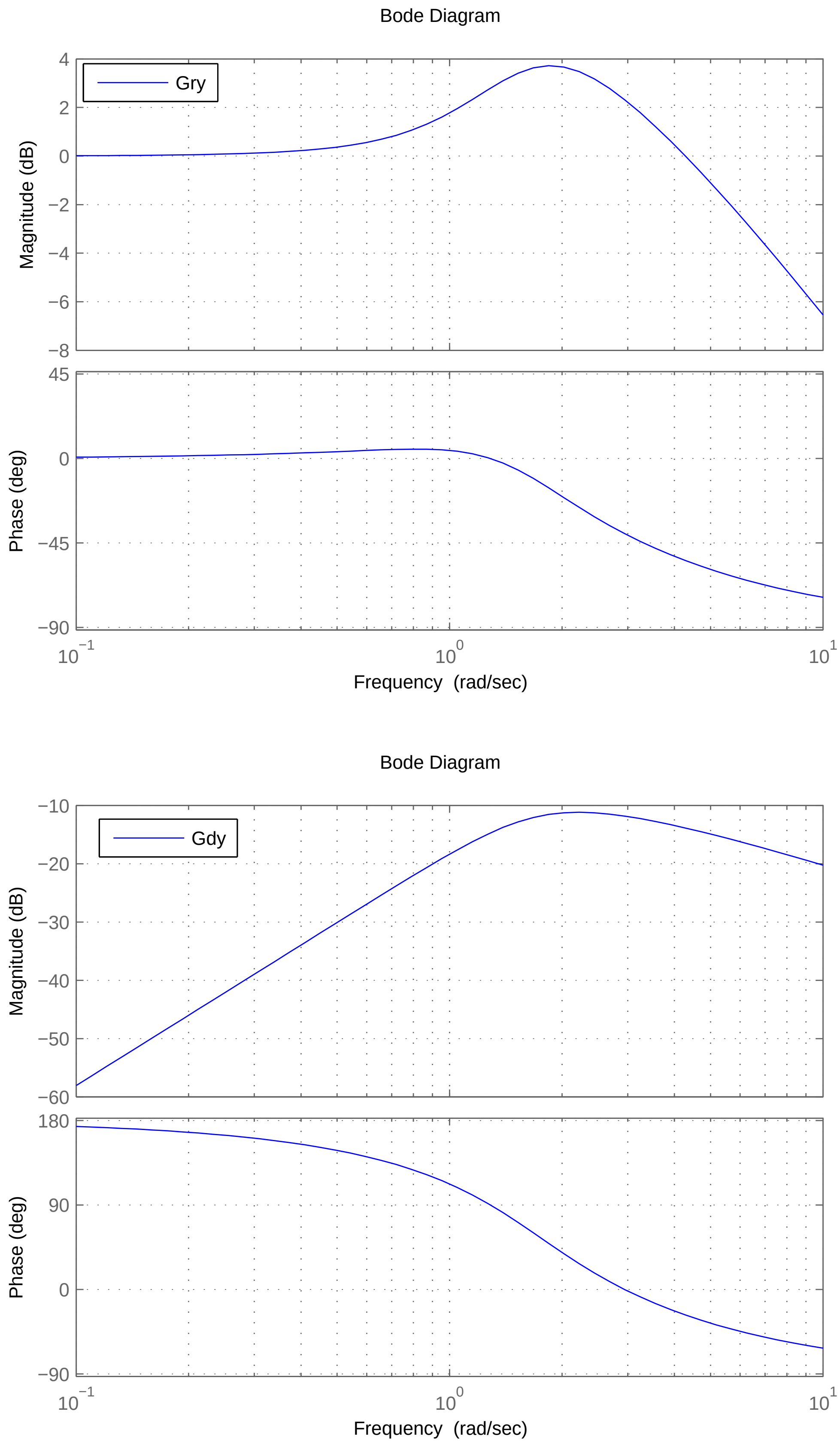

Figure 4: Magnitude Bode plot of $G_{R\to Y}(j\omega)$ and $G_{D\to Y}(j\omega)$ in problem 3.